CONTROL AND STABILITY ANALYSIS OF AN AUTONOMOUS HELICOPTER

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ABSTRACT

This paper presents some results from the research on autonomous helicopter control conducted in the framework of the COMETS project. The paper presents both linear and non-linear control laws. A two-time scale decomposition of the helicopter dynamic has been used to analyse the dynamic behaviour of the system. The fast subsystem copes with the rotational dynamics, while the slow subsystem represents the translational dynamics. The stability of the fast dynamics is demonstrated by means of a Lyapunov function. Furthermore, a feedback linearization technique is proposed to stabilize the slow dynamics. Moreover, the drawbacks of the linear control laws are pointed out and a new nonlinear control law is proposed. This control law is able to control the helicopter when large variations occur in the orientation angles and position of the helicopter.

KEYWORDS: Autonomous helicopter control, stability analysis, two-time scale analysis, Lyapunov stability, feedback linearization, nonlinear control.

1. INTRODUCTION

The work presented in this paper has been developed in the framework of the COMETS project funded by the European Commission in the IST Programme. The main objective of COMETS is to design and implement a distributed control system for the coordination of multiple heterogeneous Unmanned Aerial Vehicles (UAVs) with different autonomy degree. Both helicopters and airships are considered. The project involves the design and implementation of control techniques for autonomous helicopters and airships. This paper is devoted to autonomous helicopter control.

The methods for autonomous helicopter control can be roughly divided in learning and pilot knowledge-based control methods, and model based control methods [1]. This paper relies in the second group but also uses heuristic knowledge in the definition of the control laws. Different methods for model-based autonomous helicopter control have been presented in the literature including linear robust control based on high order linear models [2], linear control with fuzzy gain-scheduling [3], and nonlinear model predictive control [4]. In [5], linear and nonlinear control techniques are compared. It should be noted that in hovering, the nonlinear system can be linearized and then multivariable linear control techniques, such as LQR and H∞, can be applied. On the other hand, nonlinear control techniques are more general and cover wider ranges of flight envelopes but requires accurate knowledge about the system and are sensitive to model disparities. In this paper the application of both linear and nonlinear control techniques is also discussed.

The second section of the paper presents the model that has been used. Then, in section 3, a linear control law is suggested as a first control method. In Section 4 the stability of the system is analyzed by using a two-time scale decomposition, and a feedback linearization is proposed to stabilize the slow subsystem around the equilibrium point. Section 5 points out the drawbacks of

the linear control strategy and proposes a new nonlinear controller. The last two sections are devoted to the Conclusions and References.

2. HELICOPTER MODEL

Different models of the autonomous helicopter have been used in the COMETS project. In [6] the results of identification experiments are presented. The model presented in [7] has been used in this paper. The helicopter is considered as a rigid body incorporating a force and moment generation process. The connections between subsystems and state and control variables are defined in Figure 1.
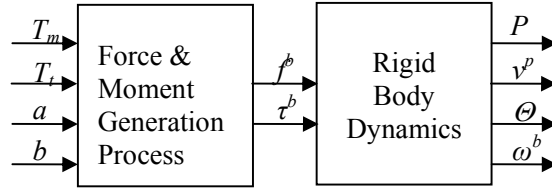

Figure 1. Connections between subsystems and state and control variables in a model helicopter.

In this model, state variables and input signals are the following (see *Figure 1*):

$$q = \begin{bmatrix} P & v^p & \Theta & \omega^b \end{bmatrix}^T = \begin{bmatrix} x & y & z & v_x^p & v_y^p & v_z^p & \phi & \theta & \psi & \omega_1^b & \omega_2^b & \omega_3^b \end{bmatrix}^T \quad (1)$$

$$u = \begin{bmatrix} T_m & T_t & a & b \end{bmatrix}^T \quad (2)$$

where P is the helicopter position in inertial coordinates, and $\Theta = [\phi\,\theta\,\psi]^T$ are the helicopter Euler angles. The forces (f^b) and torques (τ^b) generated by the main rotor are controlled by the main rotor thrust (T_m) and the longitudinal (a) and lateral (b) tilts of the tip path plane of the main rotor with respect to the shaft. The tail rotor is considered as a source of pure lateral force and anti-torque, which are controlled by the tail rotor thrust (T_t). Definition for both inertial and body coordinate frames are shown in Figure 2.
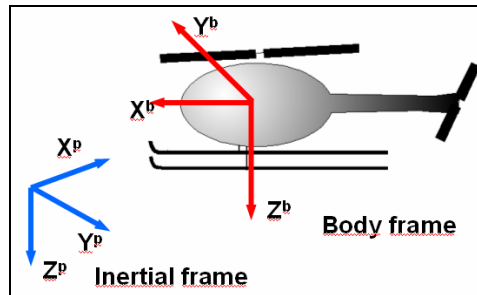

Figure 2. Inertial coordinate system and body coordinate system.

The equations of motion for a rigid body subject to body force f^b and a torque τ^b applied at the center of mass and specified with respect to the body coordinate frame is given by the *Newton-Euler* equation in body coordinate, which can be written as:

$$\begin{bmatrix} mI & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \dot{v}^b \\ \dot{\omega}^b \end{bmatrix} + \begin{bmatrix} \omega^b \times mv^b \\ \omega^b \times I\omega^b \end{bmatrix} = \begin{bmatrix} f^b \\ \tau^b \end{bmatrix} \quad (3)$$

where v^b is the body velocity vector, ω^b is the body angular velocity vector, m specifies the mass, I is an identity matrix and I is an inertial matrix. Let $R(\Theta)$ be the rotation matrix of the body axes

relative to the inertial axes (superscript p). By using the fact that $v^p = R(\Theta)v^b$ and $\Theta = \Psi(\Theta)\omega^b$, the motion equations of a rigid body can be rewritten as:

$$\begin{bmatrix} \dot{P} \\ \dot{v}^p \\ \dot{\Theta} \\ \dot{\omega}^b \end{bmatrix} = \begin{bmatrix} v^p \\ \dfrac{1}{m}R(\Theta)f^b \\ \Psi(\Theta)\omega^b \\ I^{-1}(\tau^b - \omega^b \times I\omega^b) \end{bmatrix} \tag{4}$$

3. LINEAR CONTROL

The above model is a coupled nonlinear multivariable and underactuated system with fewer independent control actuators than degrees of freedom to be controlled. However, neglecting some coupling terms, a simplified and linearized model can be obtained. The main input-output relations of this simplified model are shown in Table 1.

CONTROL INPUT	Translation	Rotation	CONTROL INPUT	Translation	Rotation
T_m	z	-	a	x	θ
T_t	-	ψ	b	y	ϕ

Table 1. Main input-output relations

Taking into account relations in Table 1, the following linear control law has been used:

$$U = \begin{bmatrix} T_m = k_1 z + k_2 \dfrac{dz}{dt} + k_3 \int z\,dt \\ T_t = k_1' \psi + k_2' \omega_3^b \\ a = k_4 x + k_5 \dfrac{dx}{dt} + k_6 \int x\,dt + k_3'\theta + k_4'\omega_2^b \\ b = k_7 y + k_8 \dfrac{dy}{dt} + k_9 \int y\,dt + k_5'\phi + k_6'\omega_1^b \end{bmatrix}, \text{ where } \begin{aligned} v_x^p &= \dfrac{dx}{dt} \\ v_y^p &= \dfrac{dy}{dt} \\ v_z^p &= \dfrac{dz}{dt} \end{aligned} \tag{5}$$

Any linear control technique can be used to calculate appropriated values for k_i, i = 1, 2... 6 and k_i', i = 1, 2... 4. The first approach used by the authors has been the design of a LQR controller, tuned from a linearization of (4) around a hover position. The designed linear controller is only valid to stabilize the helicopter around hovering position, not being allowed large variations of the state variables.

4. STABILITY ANALYSIS AND LINEAR CONTROLLER IMPROVEMENTS

The stability analysis is carried out taking into account the decomposition of the system dynamics into two time scales. One is related to the rotational dynamics, while the other, which is much slower, corresponds to the translational dynamics. This allows the study of a dynamical system to be simplified by means of two smaller dimension subsystems which evolve into two different time-scales. It can be shown that this decomposition is possible because the mass of the helicopters produces low terms of the linear velocities in (4).

4.2 Fast Subsystem. Rotational Dynamics

This subsystem is given by:

$$\dot{\Theta} = \Psi(\Theta)\omega^b$$
$$\dot{\omega}^b = I^{-1}(\tau_f + \tau_s - \omega^b \wedge I\omega^b) \tag{6}$$

where $\tau = \tau_f + \tau_s$ (*f* for fast, *s* for slow), are control variables defined as follows:

$$\tau_f \cong K(z)_\Theta \Theta + K(z)_\omega \omega, \qquad K(z)_\Theta, K(z)_\omega < 0, \forall z$$

$$\tau_s \cong K_X X_s \tag{7}$$

It can be shown that (5) can be also represented by means of (7). It is intended to guarantee the stability in a region point defined by $(\phi, \theta) < C$ around the equilibrium; τ_s is considered small enough from the fast dynamics point of view. The following Lyapunov function has been found:

$$\Lambda = \frac{1}{2} \omega^{b^T} I \omega^b - \int \Delta\Theta K_\Theta^T (\Theta) \Psi^{-1}(\Theta) d\Theta \tag{8}$$

where $\Delta\Theta$ is the change with respect to the equilibrium state Θ_0. It can be demonstrated that this function is positive if $(\phi, \theta) < C$. Furthermore, it can be demonstrated that the derivative of this function is negative:

$$\dot{\Lambda} = \omega^{b^T} K_\omega(\Theta)\omega^b < 0, \qquad \forall \Theta \in C \tag{9}$$

Then, the stability of this subsystem can be guaranteed.

4.3 Slow Subsystem. Translational Dynamics.

It is given by:

$$\dot{P} = v^p$$

$$\dot{v}^p = \frac{1}{m} R(\Theta) f^b \tag{10}$$

The force equation can be written as:

$$f^b = \Delta F + F_0 + R(\Theta)^T m \vec{g}, \tag{11}$$

where $F_0 = -R(\Theta_0)^T m \vec{g}$ are the force in the equilibrium. Consider the following feedback control law:

$$\Delta F = R^{-1}(\Theta) K_P(z) P \tag{12}$$

where K_P *(z) are feedback gains.* Substituting (11) and (12) in (10):

$$\dot{v}^p = \frac{1}{m} R(\Theta)(\Delta F + (R(\Theta) - R(\Theta_0)m \vec{g})) \tag{13}$$

Then, at the equilibrium point $\Theta = \Theta_0$, $\omega^b = 0$, the dynamics behaviour of the slow subsystem is given by:

$$\dot{v}^p \cong \frac{1}{m} R(\Theta)(R(\Theta)^{-1} K_P(z) P)$$

$$\dot{v}^p \cong \frac{1}{m} K_P(z) P \tag{14}$$

Notice that the control law (12) corresponds to a feedback linearization.

The system is stable in $(\phi, \theta) < C$ if the gains are small enough and the matrix gain $K_P(z)$ has negative eingenvalues for all *z*.

5. NONLINEAR CONTROL

It can be shown that the above control strategies only can be applied when small variations of the state variables are allowed. If large variations of these state variables can occur different control strategies should be applied. First notice that the yaw angle must be allowed to vary from $-\pi$ to π rad. The linear control law (5) is based on Table 1, being the relations between variables shown in this table expressed in local coordinate frame. Global *x* and *y* axes will not coincide

with local x and y axes if a yaw rotation is carried out by the autonomous helicopter. To solve this problem, terms of the control actions affected by errors in these coordinates must be rotated with respect to the yaw angle:

$$\begin{pmatrix} \cos\psi & -\sin\psi \\ \sin\psi & \cos\psi \end{pmatrix} \begin{pmatrix} k_4 x + k_5 \dfrac{dx}{dt} + k_6 \int x dt \\ k_7 y + k_8 \dfrac{dy}{dt} + k_9 \int y dt \end{pmatrix} \tag{15}$$

Furthermore, position errors in x, y and z should be allowed to vary from $-\infty$ to ∞ without unstabilizing the helicopter. Regarding (5) and (15), if x or y position errors increase, control actions a and b could be too high and unstabilize roll and pitch angles (Table 1) and therefore unstabilize the helicopter. To solve this problem, a nonlinear function μ is used leading to the following nonlinear control law:

$$T_m = k_1 z + k_2 \frac{dz}{dt} + k_3 \int z dt$$

$$T_t = k_1' \psi + k_2' \omega_3^b$$

$$\begin{pmatrix} a \\ b \end{pmatrix} = \mu(\|(\phi,\theta)\|) \begin{pmatrix} \cos\psi & -\sin\psi \\ \sin\psi & \cos\psi \end{pmatrix} \begin{pmatrix} k_4 x + k_5 \dfrac{dx}{dt} + k_6 \int x dt \\ k_7 y + k_8 \dfrac{dy}{dt} + k_9 \int y dt \end{pmatrix} + \begin{pmatrix} k_3' \theta + k_4' \omega_2^b \\ k_5' \phi + k_6' \omega_1^b \end{pmatrix} \tag{16}$$

$$\mu(\|(\phi,\theta)\|) = \begin{cases} 1 \; if \; \|(\phi,\theta)\| < \delta_0 \\ 0 \; if \; \|(\phi,\theta)\| > \delta_0 \end{cases} \tag{17}$$

When ϕ and θ are small enough, the linear control law with the rotation given by (15) is applied ($\mu = 1$). However, if errors in the angles are large, $\mu = 0$ and then control actions a and b will not be affected by position errors, but will try to stabilize ϕ and θ. Interpolation between the two regions can be implemented by using fuzzy logic to compute μ. Furthermore, linear control techniques can be used to compute the gains k_i and k_i' in (16). Simulations of the behavior of the system under the effect of perturbations are shown in *Figure 3*.

$$\Delta x, \Delta y, \Delta z = 1 \; m \qquad\qquad \Delta\phi, \Delta\theta, \Delta\psi = 0.2 \; rad$$

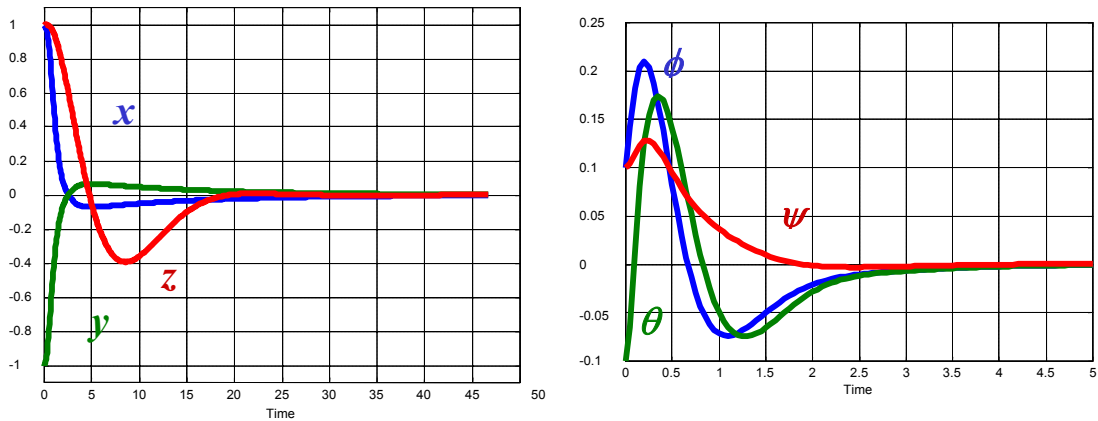**Position**   **Angles**

Figure 3. Behavior of the system under the effect of perturbations.

The stability of the system with the non-linear control law has been studied using different non-linear techniques. Particularly, the harmonic balance and the continuation method have been applied. No limit cycles of bifurcations have been found by means of these methods.

6. CONCLUSIONS

Linear control laws can be applied to control autonomous helicopters in hovering. However, if large variations of the state variables are considered, linear control techniques are not enough and can lead to instability. A two scale dynamic decomposition can be used to analyze the stability of the system. The stability of the fast subsystem (rotational dynamics) can be guaranteed by means of Lyapunov techniques. Furthermore, feedback linearization can be applied to stabilize the slow subsystem (translational dynamics) around the equilibrium point.

Non linear control techniques are useful to control the helicopter when the state variables are allowed to vary significantly around their values in the equilibrium. In this paper, a nonlinear control law which is able to control the helicopter when large variations occur in the yaw and position of the helicopter is proposed. In this control law the rotation due to the yaw is considered. Furthermore, a non-linear function precluding the loss of stability when the variations in the roll and pitch angles are significant and the helicopter is in a position significantly separated from the hovering coordinates is proposed. The application of harmonic balance and continuation methods did not detect any limit cycle or bifurcation in this nonlinear feedback system. The formal demonstration of the stability of this nonlinear control law is a future work.

7. ACKNOWLEDGEMENTS

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